Title: Stop This Runaway Truck, Please!

Brief Overview:

This unit will examine relationships between the slope of a ramp and the distance a moving object travels up the ramp.

Links to NCTM 2000 Standards:

- **Mathematics as Problem Solving, Reasoning and Proof, Communication, Connections, and Representation**
  These five process standards are threads that integrate throughout the unit, although they may not be specifically addressed in the unit. They emphasize the need to help students develop the processes that are the major means for doing mathematics, thinking about mathematics, understanding mathematics, and communicating mathematics.

  Students will demonstrate their ability to solve mathematical problems by using a model to collect and analyze data about a real-world situation. They will make and test conjectures to determine relationships between variables. Students also will express their observations about relationships in writing. In addition, they will represent a geometric situation algebraically and they will represent algebraic equations geometrically. Last of all, they will represent a real-world situation mathematically, and they will use tables, graphs, and equations to collect and organize data.

Links to Virginia High School Mathematics Standards of Learning:

- **A.2**
  The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables. Students will choose an appropriate computational technique, such as mental mathematics, calculator, or paper and pencil.

- **A.5**
  The student will analyze a given set of data for the existence of a pattern, represent the pattern algebraically and graphically, if possible, and determine if the relation is a function.

- **A.15**
  The student will determine the domain and range of a relation given a graph or a set of ordered pairs and will identify the relations that are functions.

- **G.7**
  The student will solve practical problems involving right triangles by using the Pythagorean Theorem and its converse properties of special right triangles, and right triangle trigonometry. Calculators will be used to solve problems and find decimal approximations for the solutions.

- **AII.8**
  The student will recognize multiple representations of functions (linear, quadratic, absolute value, step, and exponential functions) and convert between a graph, a table, and symbolic form. A transformational approach to graphing will be employed through the use of graphing calculators.

- **AII.19/T.19**
  The student will collect and analyze data to make predictions, write equations, and solve practical problems. Graphing calculators will be used to investigate scatter plots to determine the equation for a curve of best fit.

Grade/Level:

Grades 8-12, Algebra I, Geometry, Algebra II, Trigonometry, Pre-Calculus

Duration/Length:

2 - 90 minute blocks

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Use of Pythagorean Theorem
- Computation of slope
- Plotting coordinates

Objectives:

Students will be able to:

- collect and organize data.
- convert slope to grade of road.
- examine data in order to describe a relationship between two variables.
- select an appropriate form of a mathematical model.
- use the calculator to fit a curve to collected data.
- use a model to interpolate values.

Materials/Resources/Printed Materials:

- 2 boards (about 3 feet long and 1 foot or more wide)
- Full non-carbonated beverage can, a physics car or a heavy, free rolling toy car
- Cardboard
- Graphing calculator
- Meter sticks and rulers
- Books, magazines, cardboard to prop up boards for ramps

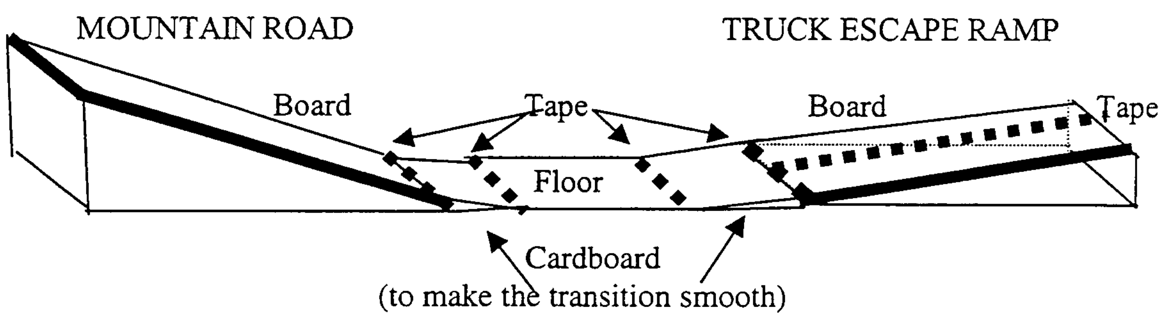
- Masking tape
- Copy of experiment directions (below in this packet)
- Lab Write Up worksheets

Development/Procedures:

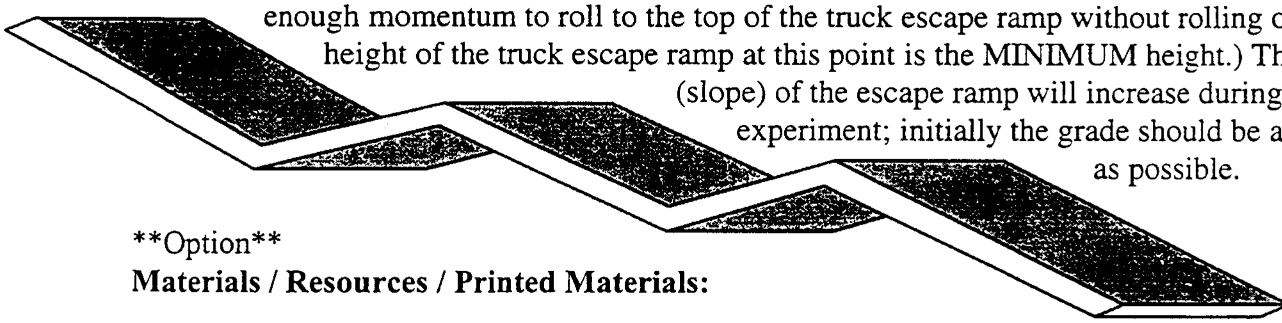
Set Up

Set up two ramps on a hard surface as shown below. One ramp represents a mountain incline; the other represents a truck escape ramp. The vehicle (can or car) will roll down the mountain ramp and up the escape ramp.

Sample set-up of ramps



Use cardboard as needed to eliminate the bumps between the end of each board and the floor. Run a strip of masking tape the entire length of the truck escape ramp. Beginning at the bottom of the ramp, mark off the tape in convenient units of length: half-inch, cm, etc. Separate the two ramps by at least one can/car length. Adjust the height of both ramps so that the can/car builds enough momentum to roll to the top of the truck escape ramp without rolling off. (The height of the truck escape ramp at this point is the MINIMUM height.) The grade (slope) of the escape ramp will increase during the experiment; initially the grade should be as small as possible.



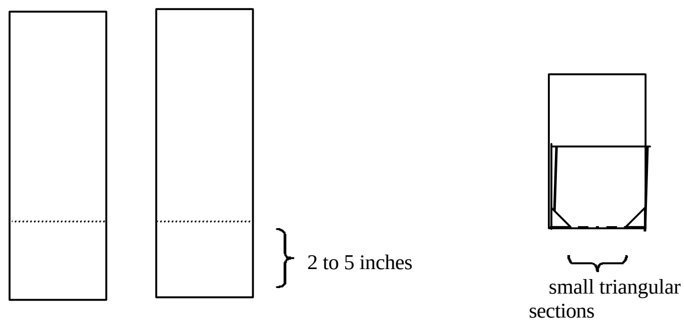
Option

Materials / Resources / Printed Materials:

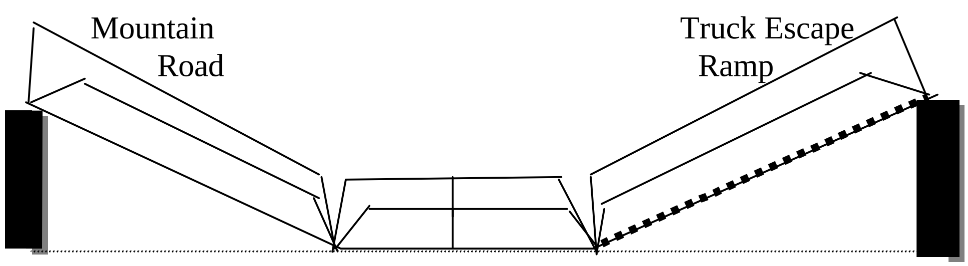
- 2 long strips of poster boards about 2 inches wide and at least 20 inches long.
- Small ball or marble
- Graphing Calculator
- Meter sticks and rulers
- Books, magazines, cardboard to prop up poster boards for ramps
- Masking tape
- Copy of directions
- Lab Write up worksheets

Set Up

One strip of poster board will be used for the mountain road the other strip for the truck escape ramp. Fold the two strips of poster board two to five inches from the bottom. Cut out two small triangular sections at each fold from both strips. Unfold the strips of poster board. Place a strip of masking tape down the center of one of the poster board strips, from the fold to the edge. This strip will be used for the truck escape ramp. From the fold, mark off the tape in convenient units of length: half-inch, cm, etc. Fold both ramps down the center.



Tape the bottoms of the two ramps together. Prop the two ramps on a hard surface as shown below. One ramp represents the mountain road; the other represents the truck escape ramp. The small ball will roll down the mountain and up the escape ramp.



Experiment Directions

Measure the following lengths:

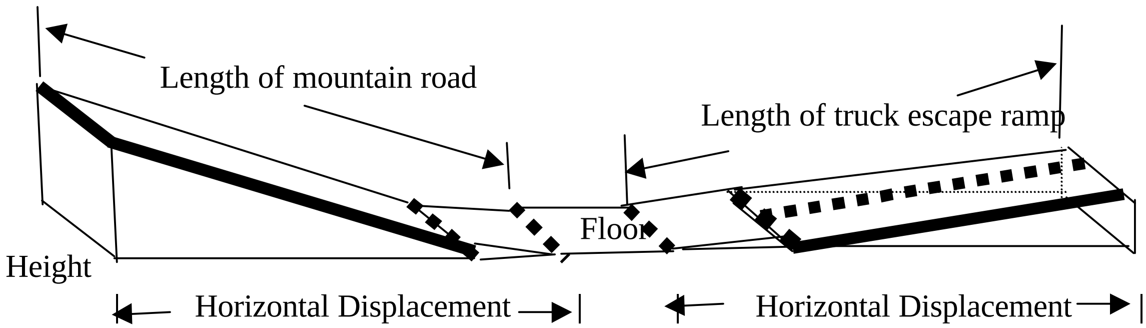
- Length of the mountain road
- Height of the mountain road
- Length of the escape ramp
- Height of the escape ramp

Use these values to compute the following measures:

- Horizontal displacement of the mountain road
- Grade of the mountain road
- Horizontal displacement of the escape ramp
- Grade of the escape ramp

Note: grade = $\frac{\textit{height}}{\textit{horizontal displacement}}$ x 100

Record the measurements and the computed values in a table.



Hold the can/car at the top of the mountain road. Release it. Observe the distance the can/car travels up the escape ramp before it rolls backward. Record this value. Repeat the procedure two more times and find the average of the three distances.

Increase ramp height by a small increment. Run the experiment again. Repeat this process to obtain at least 6 more sets of data. The height should have a range of at least 4 inches.

Assessment:

Student achievement will be based on the student lab write-up sheet.

Extension/Follow Up:

- Write to the highway department of your state to find out whether any truck escape ramps exist nearby or in your state. If there is a nearby ramp, gather information about it.
- Use a sheet of sandpaper to simulate gravel or sand. Does the sandpaper affect the stopping distance?
- Increase the length of the level grade between the two ramps. Do successive increases affect the stopping distance?

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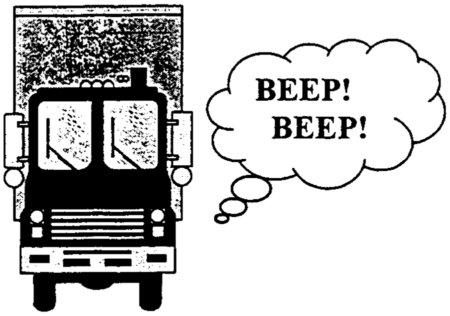
LAB WRITE-UP
STOP THIS RUNAWAY
TRUCK, PLEASE!

Name ______________________________
Date ______________________________

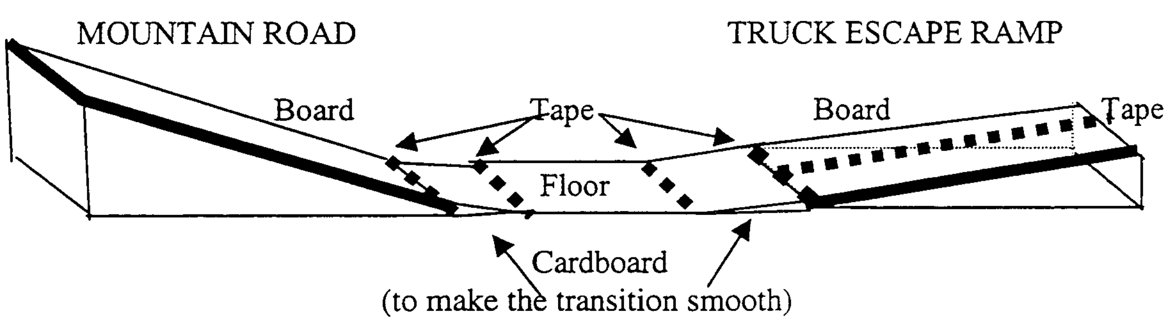


Problem: Road Runner and Coyote are up to their usual antics. This time Coyote decides to use his 18-wheeler to catch Road Runner. He remembers his last experience. *He was traveling down the mountain road at a tremendously fast speed, when all of a sudden his brakes failed. His only recourse was to crash into the side of the mountain to stop the truck.* But this time Coyote has a plan. He will build an escape ramp, so that if his brakes fail, his runaway truck will come to a safe stop. But there is a problem: he has a limited amount of room. At what grade should Coyote build his escape ramp?

<u>Experiment Set Up</u>

Set up two ramps on a hard surface as shown below. One ramp represents a mountain incline; the other represents a truck escape ramp. The vehicle (can or car) will roll down the mountain ramp and up the escape ramp.

<u>Sample set-up of ramps</u>



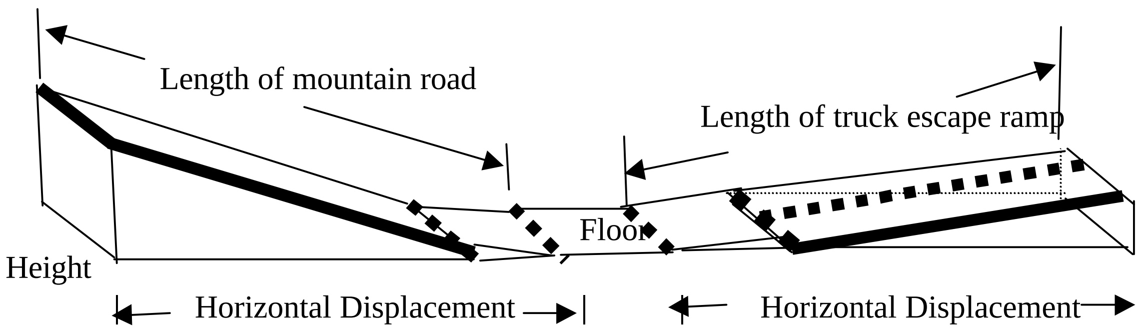
Use cardboard as needed to eliminate the bumps between the end of each board and the floor. Run a strip of masking tape the entire length of the truck escape ramp. Beginning at the bottom of the ramp, mark off the tape in convenient units of length: half-inch, cm, etc. Separate the two ramps by at least one can/car length. Adjust the height of both ramps so that the can/car builds enough momentum to roll to the top of the truck escape ramp without rolling off. (The height of the truck escape ramp at this point is the MINIMUM height.) The grade (slope) of the escape ramp will increase during the experiment; initially the grade should be as small as possible.

Experiment Directions

Measure the following lengths:

- Length of the mountain road
- Height of the mountain road
- Length of the escape ramp
- Height of the escape ramp

Record the measurements in a table. (See Lab Write Up, # 4)



Hold the can/car at the top of the mountain road. Release it. Observe the distance the can/car travels up the escape ramp before it rolls backward. Record this value. Repeat the procedure two more times and find the average of the three distances.

Increase ramp height by a small increment. Run the experiment again. Repeat this process to obtain at least 8 more sets of data.

1. What is the purpose of the experiment? (Why are we doing this?)

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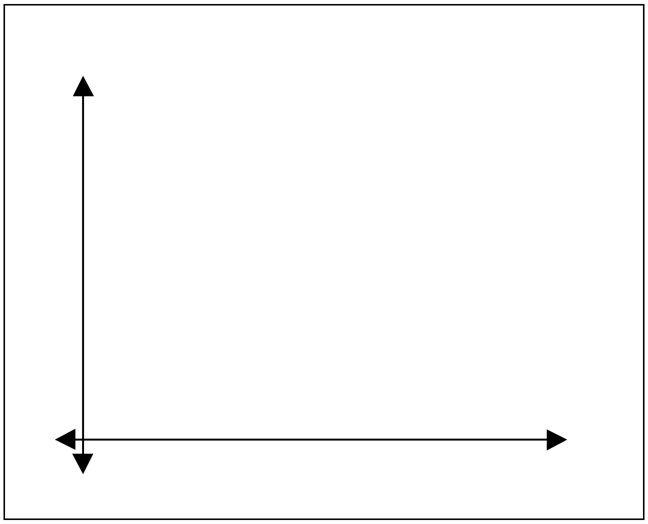
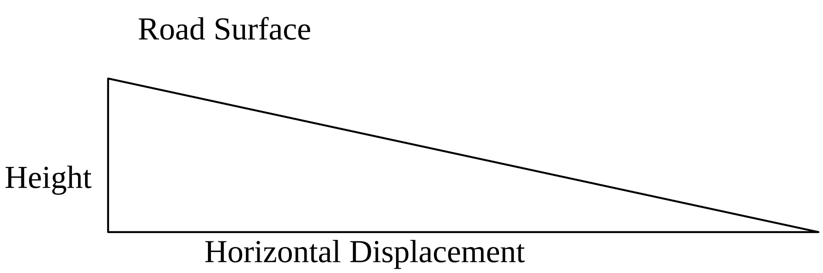
2. Before you collect your data, make a conjecture about this experiment. What do you think the graph of your data would look like? Predict the grade of Coyote's escape ramp. Explain your reasoning.



3. Sample problem: Understanding the grade of a road. The grade of a road is

   $\dfrac{\textit{height}}{\textit{horizontal displacement}}$ x 100. The grade of a road is usually expressed as a percent.

   Consider for example, a 5-mile section of highway that rises (or drops) 1850 feet. What is the grade of this stretch of highway? The sketch below is not drawn to scale.



Five miles = 26,400 feet. Height = 1850 ft. Use Pythagorean Theorem to find horizontal displacement. $26400^2 = 1850^2 +$ horiz. dis.^{2}, therefore horizontal displacement = 26,335 ft. (rounded to the nearest integer), So, grade = 1850/26335 x 100 or about 7% grade.

How can we find the horizontal displacement of the ramp when we know its height and its length? ____

What two pieces of information do we need to compute the grade of the ramp?

____

4. Data Collection

Run the experiment following the directions above. Please use these tables to record your experiment data:

Setup	Length	Height	Horizontal Displacement	Grade
Mountain Road				
Escape Ramp				

Escape Ramp

Height	Horizontal Displacement	Grade	Stopping Distance			
			Trial 1	Trial 2	Trial 3	Average

5. Graph:

The independent variable represents ______________________________

The dependent variable represents _______________________________ Units______

List the elements of the domain:

List the elements of the range:

TI-83 Instructions:

Press STAT, EDIT (or #1) and enter the values from your grade column above into L1. Enter the average stopping distances from your table into L2. To view the graph of your data, press 2nd STAT PLOT and select plot 1. Select ON. Select the scatter plot (first icon). In Xlist, enter L1 and in Ylist, enter L2. Select your favorite mark. Press ZOOM 9 (ZoomStat).

How does this graph compare to the graph you drew in Question #2?

Please copy the graph from your calculator onto the graph below. Be sure to label your axes with your independent and dependent variables. Indicate the scale for each axis.

6. Data Analysis:

a. Describe what you see on your graph. Is the data linear?

b. What do your data and graph tell you about the physical situation?

c. Does it make sense to connect your points with a smooth curve? Explain.

d. Does it make sense to extend the curve beyond your data points? Explain.

e. Draw a smooth curve through the data points. State the new domain.

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f. Is there a x-intercept? ☐ yes ☐ no . If yes, explain its relationship to the physical situation. If no, explain why not, in terms of the physical situation.

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g. Is there a y-intercept? ☐ yes ☐ no . If yes, explain its relationship to the physical situation. If no, explain why not, in terms of the physical situation.

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7. What type of function might the curve represent? ______________________________

8. Select a grade between your minimum and maximum escape ramp grades. Complete the table below using your curve to predict the corresponding stopping distance.

Grade	Stopping Distance

9. Use half of the escape ramp length as your stopping distance. Complete the table below using your curve to predict the corresponding grade.

Stopping Distance	Grade

10. Coyote will succeed for the first time because of your input. What advice would you give him?

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****Extension for more advanced classes *****

11. Consider all of the families of curves that you are familiar with. Choose a family of curves and try to find an equation that closely fits your data. You may use your calculator.

    What family of curves did you choose?___________________________________

    To find the regression equation using your TI-83, press STAT, CALC. Select any model 5 or below to C and push ENTER. If you like, do this for each type of equation. Compare the regression coefficients to see which equation provides the best fitting curve.

    Equation____

12. Are you satisfied with your equation? Do you think you could have found a better fitting model? Explain.

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    ____

    ____

13. How would your graph change if you use height as the independent variable?

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    ____

14. How would your graph change if you use angle measure as the independent variable?

    ____

    ____

    ____

Teacher Notes

Background:

Steep or particularly long downgrades challenge the drivers of heavy trucks. If brake failure occurs on such a road, the runaway truck poses a tremendous danger to the truck driver, other drivers on the road, and people in roadside businesses and homes. Truck escape ramps (TERs), which have been part of our highway system for more than 30 years, have been highly effective in reducing traffic fatalities, injuries, and property damage in runaway truck incidents. Two main design features are used in the construction of TER's: loose gravel or sand and a ramp built on an upgrade. This unit attempts to model the effect of the upgrade on the stopping distance.

References:

"Truck Escape Ramps: Determining the Need and the Location," Road Management & Engineering Journal, TranSafety, Inc., August 10, 1997.

The following Internet site provides an annotated bibliography of 33 publications on this topic. Most of the publications are reports written under contract to various state highway departments and/or the Federal Highway Administration:

http://www.pascousa.com/febs/trb/truckesc.ref.txt

The concept of the grade of a road, which will be unfamiliar to many students, should be developed before students begin the lab write up. Some students may think that grade is measured in degrees (not true). You may wish to begin the explanation by asking whether students have ever seen road signs warning motorists of a steep grade. Perhaps they remember seeing grade expressed as a percent. The grade is simply $\frac{\textit{height}}{\textit{horizontal displacement}} \times 100$.

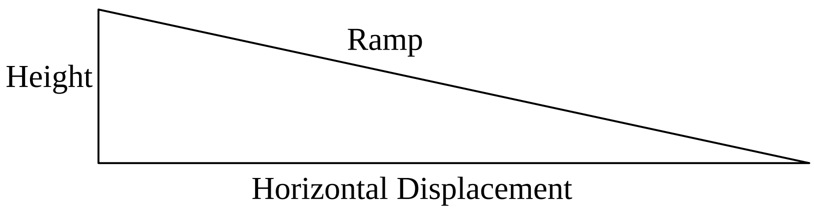
This implies a slope of 1 (45-degree angle with horizontal) corresponds to a 100% grade; a slope of 1/2 (27-degree angle with horizontal) corresponds to a 50% grade. Students should realize that these are not realistic values for the grade of a road! Truck escape ramps are typically built on highways with a grade between 5% and 8%. A few highways may have a grade as steep as 10%, but the length of such a steep grade will be fairly short: .3 to .5 mile.

When the students find the horizontal displacement of the escape ramp, they will find that it is only slightly less than the ramp length. Discourage them from rounding-off this number; they may round off when they compute the grade.

Sample Data

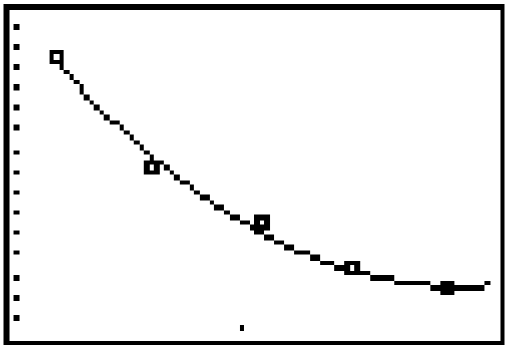
Beginning of escape ramp located three feet from end of down ramp
Escape ramp length constant at 39.625 inches
Down ramp length constant at 34 inches

Height of escape ramp (in)	Horizontal length beneath escape ramp (in)	Grade of road	Stopping Distance (in)
1.500	39.597	3.8%	39.5
1.750	39.586	4.4%	34.2
2.000	39.574	5.1%	31.5
2.250	39.561	5.7%	29.3
2.500	39.546	6.3%	28.4



Graph from sample data points on a TI-83:

Stopping Distance vs. Grade of Truck Escape Ramp



Answers to Lab Write Up

1. To find the right grade for Coyote's ramp. Some advanced students may state that the purpose of the experiment is to determine the relationship between the grade of the road and the stopping distance.
2. Answers will vary. Any graph that shows stopping distance decreases, as grade increases, should be considered correct at this step. This is a good time to develop the concept of grade of a road. Any number greater than 0% and less than 25% should be acceptable at this stage.
3. A. Use the Pythagorean Theorem.
   B. We need the height of the ramp and the horizontal displacement.

4. Data values will vary. See teacher notes for sample data set. When the students find the horizontal displacement of the escape ramp, they will find that it is only slightly less than the ramp length. Discourage them from rounding off this number; they may round off when they compute the grade.
5. The independent variable is grade of road. The dependent variable is stopping distance. The elements of the domain are listed under grade in the student's data table, and the elements of the range are listed under average stopping distance in the student's data table. A sample graph is provided in teacher notes, although graphs may vary based on student data. Students' comparisons of their graphs from question #2 and question #5 will vary.
6. a. The graph shows a series of points for which the y value decreases as the x value increases. The data are not linear.
   b. The graph tells us that for a constant increase in grade, there is a decrease, but not a constant decrease, in stopping distance. Greater decreases in stopping distance occur when the grades are low.
   c. Yes. The grade and stopping distance can have any value between the maximum and minimum domain/range values. Grade and stopping distance are continuous variables.
   d. It does not make sense to extend the curve to the left of the first plotted point because greater stopping distance represents overrunning the ramp and crashing into a barrier. It does make sense to extend the curve a short distance to the right of the last plotted point, but there is a limit to the grade of the ramp. If the ramp is too steep, the driver will lose control of the vehicle. This provides an opportunity to distinguish between interpolation and extrapolation. The latter is almost always a dangerous exercise.
   e. See sample graph in teacher notes. Graphs may vary. The new domain is all real numbers between the minimum and maximum grade listed in the discrete domain in question 6.
   f. No. Stopping distance of zero makes no sense in this problem.
   g. No. When the grade of the ramp is zero, there is a finite stopping distance due to friction. (Actual escape ramps use gravel and/or sand to stop the trucks.) But, in the context of this problem, any stopping distance greater than the ramp length corresponds to a crash.
7. The data may be modeled by a quadratic. Some students may choose a parabola and others may choose a hyperbola. See additional comments under answers to extension problems.
8. Answers will vary.
9. Answers will vary. Some students may have to extrapolate to answer this question.
10. Answers will vary. Students should find the point where additional increases in grade offer very small decreases in stopping distance.

****Answers to extensions****

11. Some students may choose a parabola, others a hyperbola. Regression analysis using the TI-83 may lead students to a parabola, but some students may discover that the power regression gives the *best*-fit curve. (Some students may find that a cubic or a quartic equation fits better, but they need to understand that these models don't match the physics of this problem. Equations will vary.
12. Answers will vary.
13. The new graph is a transformation of the original graph. One observes a shift and a stretch.
14. The new graph is a transformation of the cotangent function.